PHYS 231 – Assignment #1

Due Wednesday, Oct. 4 at 10:00 am

At the beginning of term we obtained the important results given below.

Standard Deviation and Standard Error

If you measure a quantity y numerous times the measurements will be Gaussian distributed. The width of the distribution is determined by the standard deviation σ which is given by:

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (y_i - \overline{y})^2}, \tag{1}$$

where N independent measurements of y are made and:

$$\overline{y} = \frac{1}{N} \sum_{i=1}^{N} y_i, \tag{2}$$

is the average of value of the N measurements. Now suppose that you repeat the N measurements of y many many times and determine $\langle y \rangle$ each time. The values of $\overline{y}$ will themselves be Gaussian distributed with a width given by:

$$\sigma_{\overline{y}} = \frac{\sigma}{\sqrt{N}}. \tag{3}$$

$\sigma_{\overline{y}}$ is called the standard error and is the experimental uncertainty that should be reported when you make N measurements of quantity y $\Rightarrow$ final result of the N measurements is $\overline{y} \pm \sigma_{\overline{y}}$.

Error Propagation

Suppose that some function f is dependent on the variables x_1, x_2, ..., and x_N such that $f = f(x_1, x_2, \ldots, x_N)$. If we make measurements and estimate the uncertainties of the N variables $x_1 \pm \Delta x_1$, $x_2 \pm \Delta x_2$, ..., and, $x_N \pm \Delta x_N$, then we can determine the uncertainty Δf in f using the error propagation equation:

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\Delta x_1\right)^2 + \left(\frac{\partial f}{\partial x_2}\Delta x_2\right)^2 + \cdots + \left(\frac{\partial f}{\partial x_N}\Delta x_N\right)^2}. \tag{4}$$

Here, the "curly d's" represent the partial derivatives of f with respect to x_i. It means that, when evaluating $\partial f/\partial x_i$, you should treat all the variables of function f as constants except for x_i.

Assignment

1. Find the uncertainty Δy in y as a function of the uncertainties Δu and Δv in u and v for the following functions:

(a) $y = \dfrac{1}{2(u+v)}$

(b) $y = uv^2$

2. The initial activity N_0 and the lifetime τ of a radioactive source are known with uncertainties of 1% each. The activity at time t is given by $N = N_0 e^{-t/\tau}$. Assume that the uncertainty in t is negligible. Confirm that the error in activity ΔN is dominated by the ΔN_0 term for $t \ll \tau$ and by $\Delta \tau$ term for $t \gg \tau$. For what value of t/τ do the errors in N_0 and τ contribute equally to the uncertainty in N?

This second portion of the assignment is used as review of concepts covered in PHYS 102/121/122 (or an equivalent course).

3. Find the current in R. *Hint: Look for the most convenient loop.*

Res-Monster Maze

All batteries are 4V
All resistors are 4Ω

Find current in R
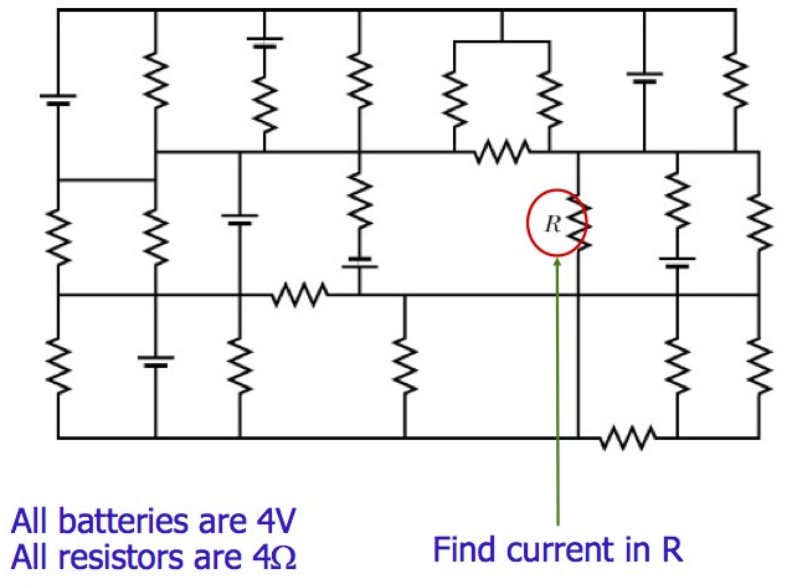
Also complete and submit for grading the following problems from your textbook:
Eggleston Chapter 1: 6, 16

Additional Practice Problems

– These problems won't be graded, check your answers in the back of the book

Eggleston Chapter 1: 3, 5, 7, 9, 11, 15
Eggleston Chapter 2: 1, 3

More Practice Problems!

1. Find the uncertainty Δy in y as a function of the uncertainties Δu and Δv in u and v for the following functions:

(a) $y = \dfrac{1}{2(u-v)}$

(b) $y = \dfrac{1}{u^2}$

(c) $y = u^2 + v^2$

2. If the diameter of a round table is determined to within 1%, how well is its area known? Would it be better to determine its radius to within 1%?

3. The figure below shows a pair of springs with spring constants k_1 and k_2 connected in (a) series and (b) parallel. One end of each combination is attached to a rigid wall and the opposite end to a block of mass m. A force F is applied to each system as shown in the figure.
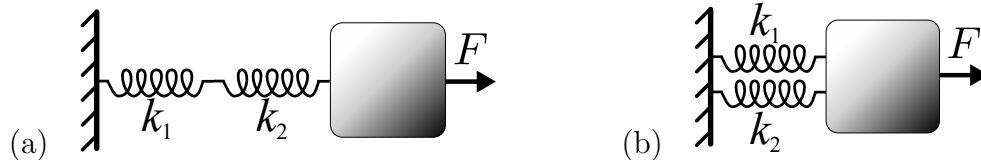
(a) Determine the equivalent spring constant of the series combination of springs.
(b) Determine the equivalent spring constant of the parallel combination of springs.

Show all of your work.

4. Snell's law relates the angle of refraction θ_2 of a light ray travelling in a medium of index of refraction n_2 to the angle of incidence θ_1 of a ray travelling in a medium of index of refraction n_1 through the equation $n_2 \sin \theta_2 = n_1 \sin \theta_1$. Find n_2 and its uncertainty from the following measurements:

$$\theta_1 = (22.03 \pm 0.2)^\circ \quad \theta_2 = (14.45 \pm 0.2)^\circ \quad n_1 = 1.0000$$

Assume that the uncertainty in n_1 is negligible.

5. After measuring the speed of sound u several times, a student concludes that the standard deviation of her measurements is $\sigma_u = 12$ m/s. Assume that the uncertainties are random and that the experiment is not limited by systematic effects. Determine how many measurements would be required to give a final uncertainty in the mean of ± 2.0 m/s.